Optimization Modeling in Python: Fundamentals and a Kidney Exchange Example

Because of our new budget we need to lower your spendings by 40%

Vidit | MS Biostatistics ‘26

Plan for today:

1) Feasible vs Infeasible
2) Basic Problem Structure
3) Pulp code (General)
4) Focus on matching: Kidney exchange
5) Pulp code for toy Kidney exchange Data

Nevermind, I will figure something else out to meet the budget.

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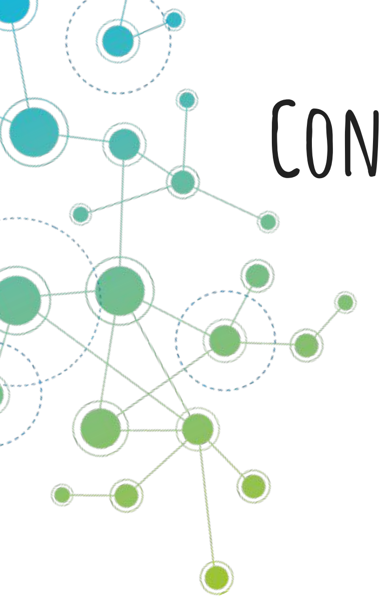
Feasibility

Constraints

In every optimization problem we have some constraints. Examples:

1) Positive values only
2) Storage capacity
3) Budget
4) Time
5) Distance

… etc.



Constraints: Example

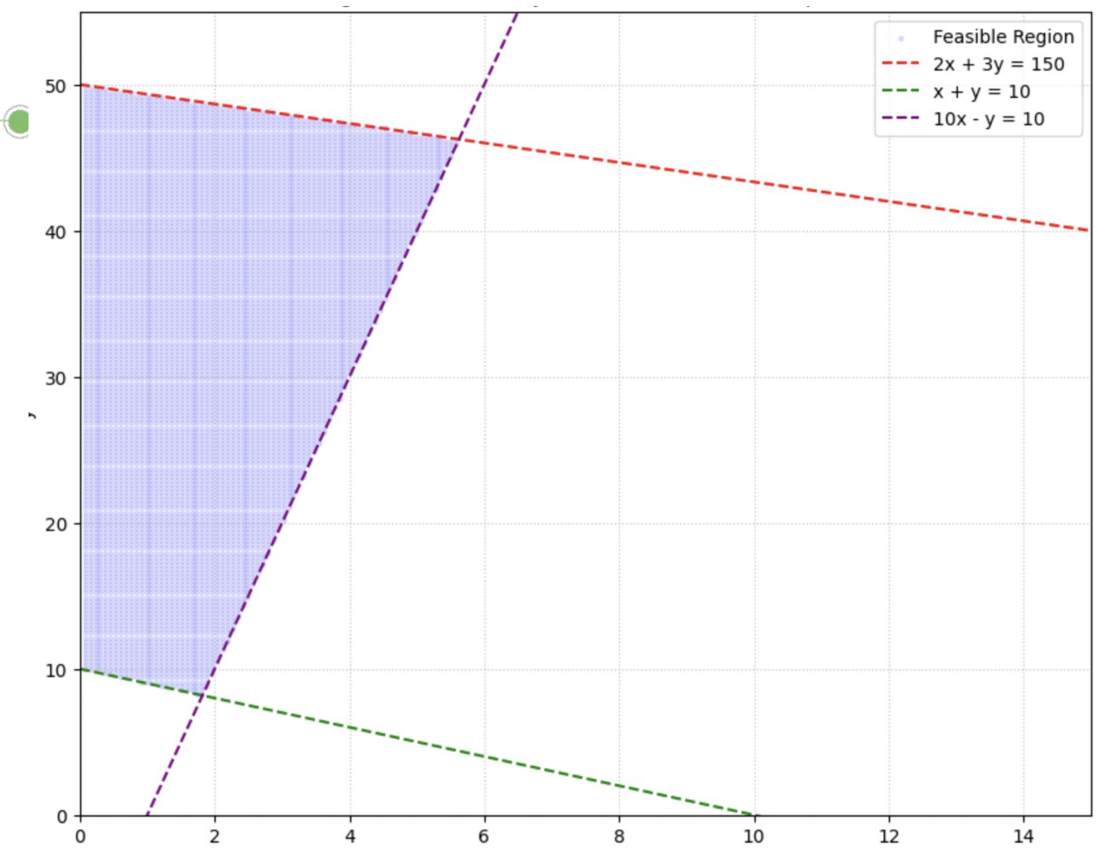
- Has only 1 oven at home: can’t produce over a certain limit in one day.
- Doesn’t have unlimited space to store resources like eggs, oils, flour etc.
- Refrigerating capacity … etc.

Big dreams but a small purse with little money don’t go very far: Budget constraint

Feasible

A problem is feasible if there exists a solution/ solutions that satisfy all the constraints. Such a solution is a feasible solution. If no feasible solution exists then the problem is infeasible.

Feasible Region (2 Variables)



1) 2x + 3y <= 150

2) x + y >= 10

3) 10x – y <= 10

4) x>= 0

5) y >= 0

Feasible Region (3 Variables)

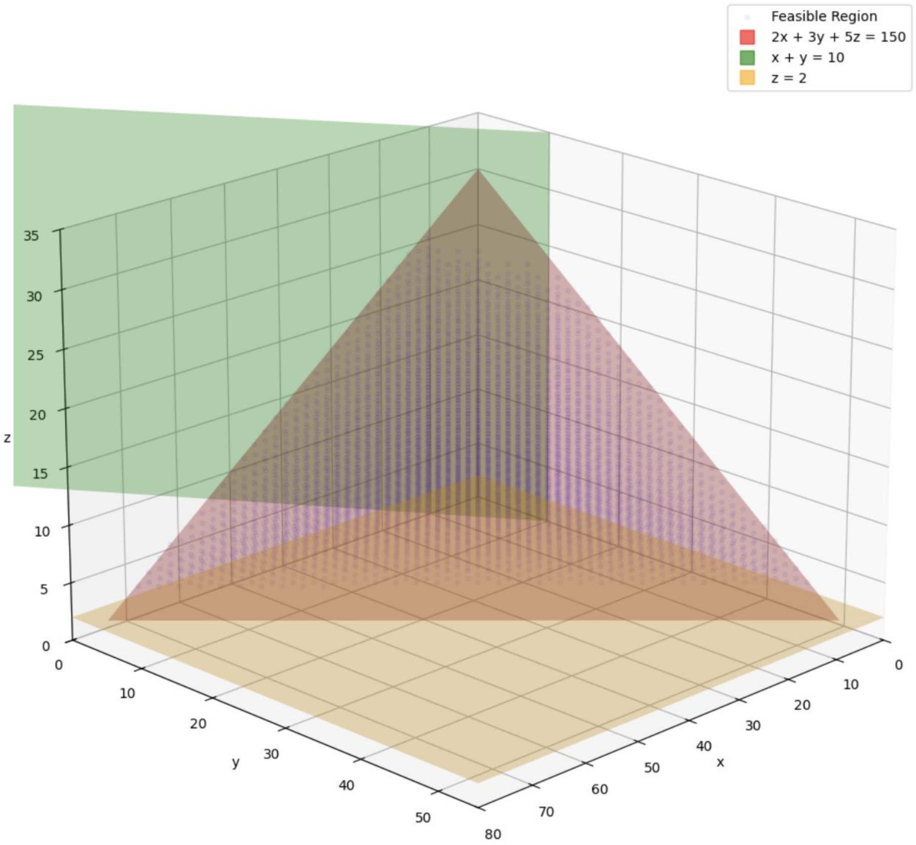
1) 2x + 3y + 5z <= 150

2) x + y >= 10

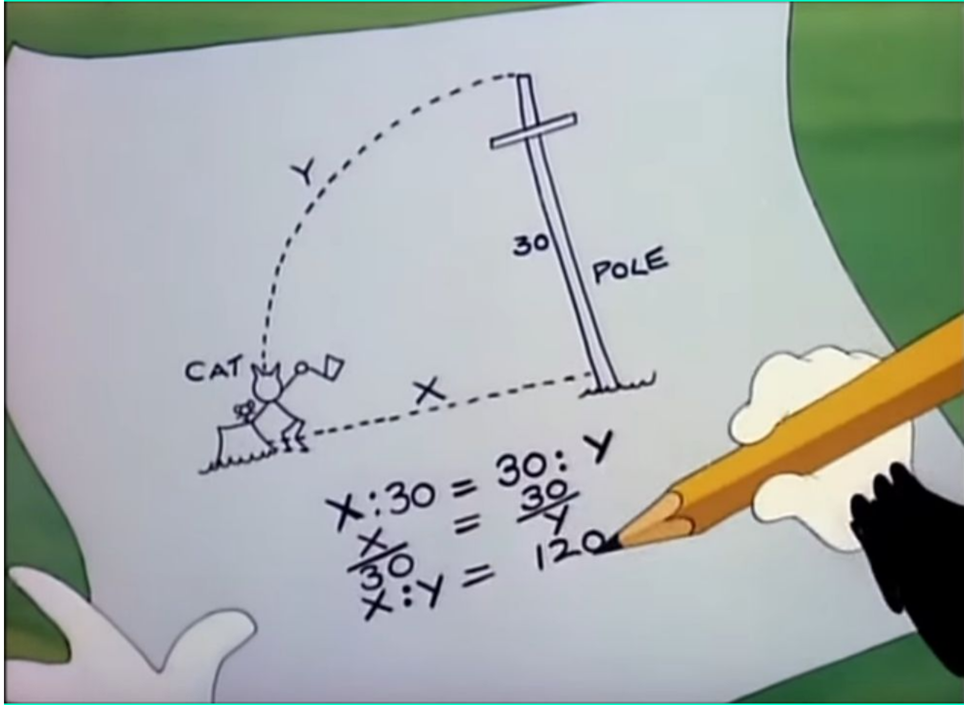
3) z >= 2

4) x>= 0

5) y>= 0



Basic Structure Of Problems



Objective Function

Given feasible solutions, you want to pick the best one.

This is the optimal solution(s): we choose the solution(s) that maximize or minimize the objective function.

Example: Minimize: $f(x,y) = 3x + 5y$

Objective Function

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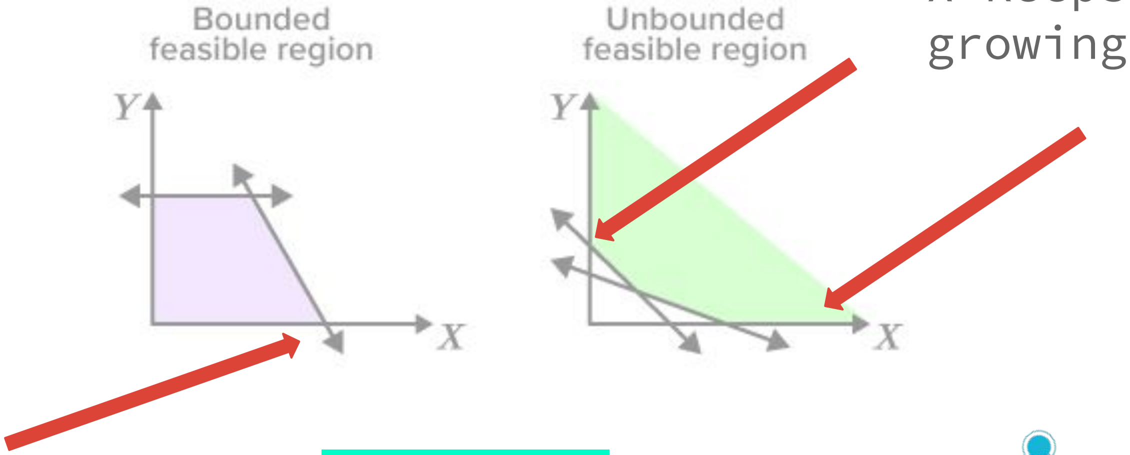
Feasible Region (Bounded vs Unbounded)



X keeps growing

X cannot grow beyond this

Maximize X

Linear Programming

Linearity: Objective function and constraints are straight lines i.e. no quadratic, exponential etc. terms.

Corner Point Property: At least one corner point is the/ one of the optimal solutions if an optimal solution exists.

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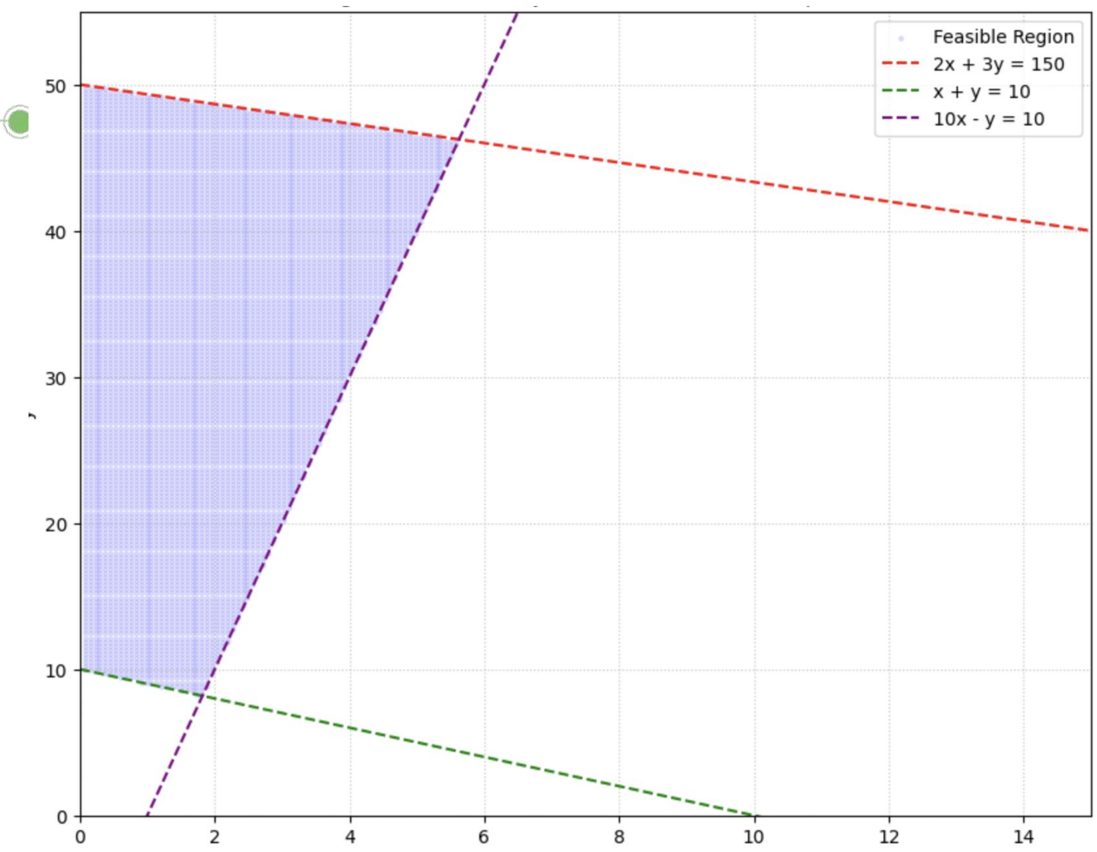
Corner Points Example

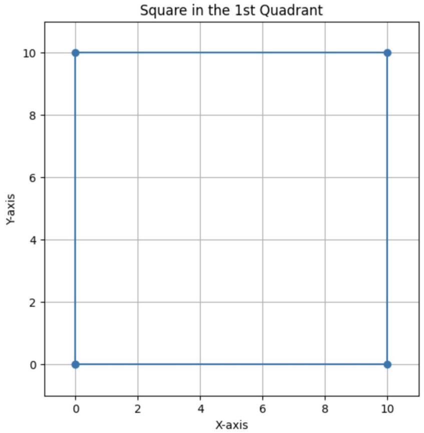
(0,10)

(0, 50)

(20/11, 90/11)

(45/8, 185/4)



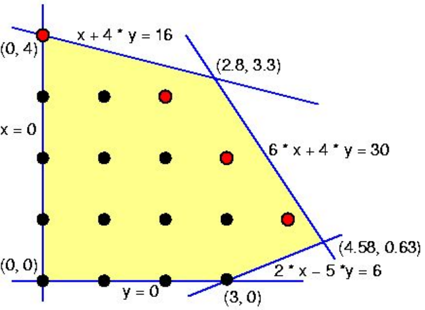


Integer (Linear) Programming

Linearity: Objective function and constraints are straight lines i.e. no quadratic, exponential etc. terms.

Values have to be Integers

Corner Point Property: No longer maintained



Function to maximize: f(x, y) = 6 * x + 5 * y
Optimum LP solution (x, y) = (2.4, 3.4)
Pareto optima: (0, 4), (2, 3), (3, 2), (4, 1)
Optimum ILP solution (x, y) = (4, 1)

© Graph in this page from Duke, CS

Other Differences b/w LP and IP

- In LP the feasible region is a convex polyhedron. In IP the region is harder to visualize- scattered points inside polyhedron.
- LP is solvable in polynomial time. IP is NP-hard.
- Taking integer constraints away from IP gives it’s LP relaxation.
- IP is often times more realistic.
- Moving along edges VS Branch and Bound

Mixed Integer (Linear) Programming

Linearity: Objective function and constraints are straight lines i.e. no quadratic, exponential etc. terms.

Values are a mix of Continuous and Integers

Corner Point Property: N/A

More the continuous variables the more it is faster for solver to solve usually.

Multi-objective Optimization

More than one objective function.

For example: We want to maximize profit but minimize labor cost.

Improving one objective worsens another.

Different solutions

PULP

Some Pulp
Tropicana
LIGHT
50% LESS SUGAR & CALORIES THAN OJ
100% DV of Vitamin C

Basic Information

PuLP (Python Universal Linear Programming) is an open-source Python library that simplifies constructing and solving linear optimization models. It provides a user-friendly syntax for defining variables, objective functions, and constraints, making LP accessible to users with varying programming backgrounds. It is widely used in various industries such as Finance and Supply Chain.

Source: https://medium.com/operations-research-bit/diving-into-pulp-a-pythonic-approach-to-linear-programming-7b20ca15aa88

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Examples Using PuLP

Important: PuLP is the modeling language. Internally, a solver (optimization engine) computes the solution.

Now we will look at some simple examples and how we can "solve" them using PuLP.

Examples Using PuLP (Scenarios)

Checking Feasibility

Scenario 1: LP- Metals for Utensil Production (also used for MILP)

Scenario 2: IP- John’s Pizza: How many to produce?

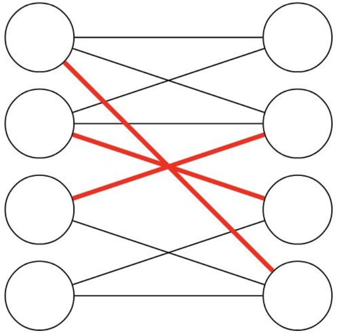
Scenario 3: Multiobjective- Blood Drive: Maximize Donations + Minimize Cost

Kidney Exchange

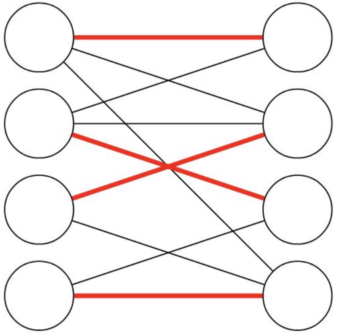
Matching Problems

Goal: Maximum matching; assign as many participants as possible (without conflicts).

Example: Matching persons to jobs etc.



Imperfect



Perfect

Kidney Matching Intuition

Suppose X needs a kidney.

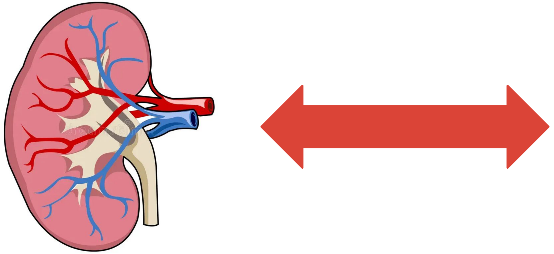
X’s friend is willing to donate. But they are not compatible.

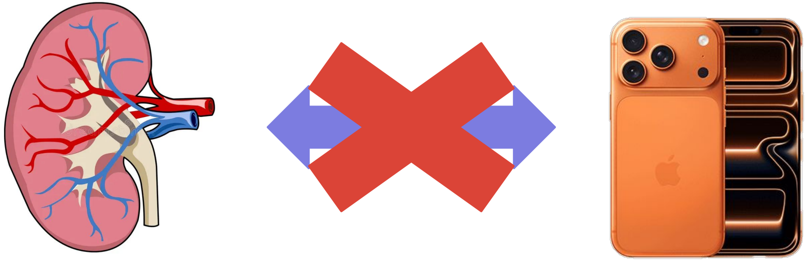
Suppose Y has a similar situation: needs kidney- friend can donate- incompatible.

If X’s friend is compatible with Y and Y’s friend is compatible with X
they can “exchange” donors.

Kidney Matching Goal

Goal: Match as many of these patient-donor pairs so that most receive a kidney.



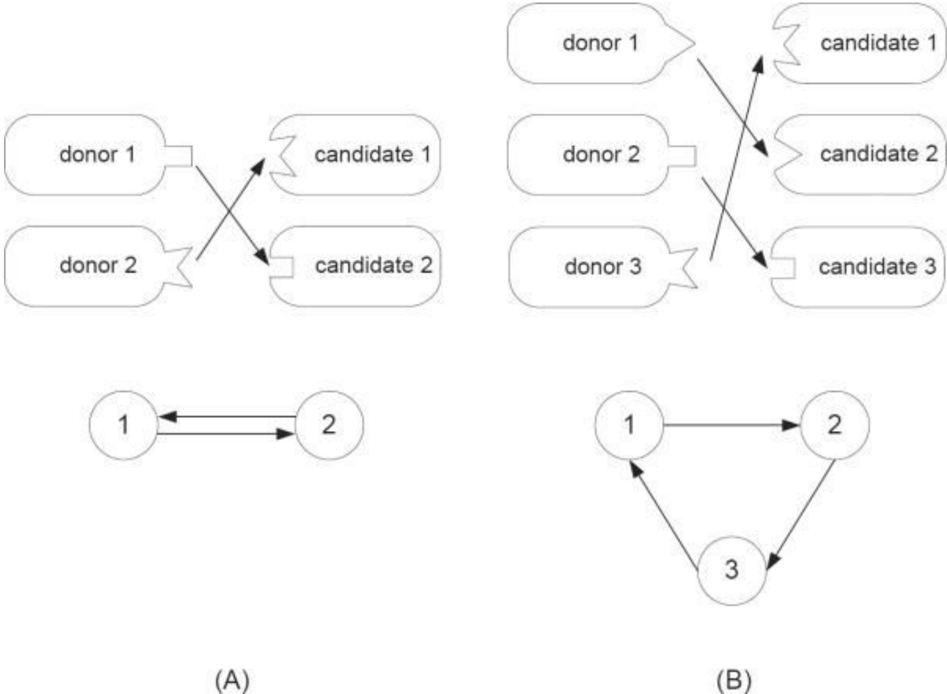


PuLP Toy Example

Toy data- We shall look at 2-way and 3-way exchanges. We will also consider giving some patient priority.

Assumptions here:

1) Only 2 and 3 way exchanges.
2) Every exchange is successful.

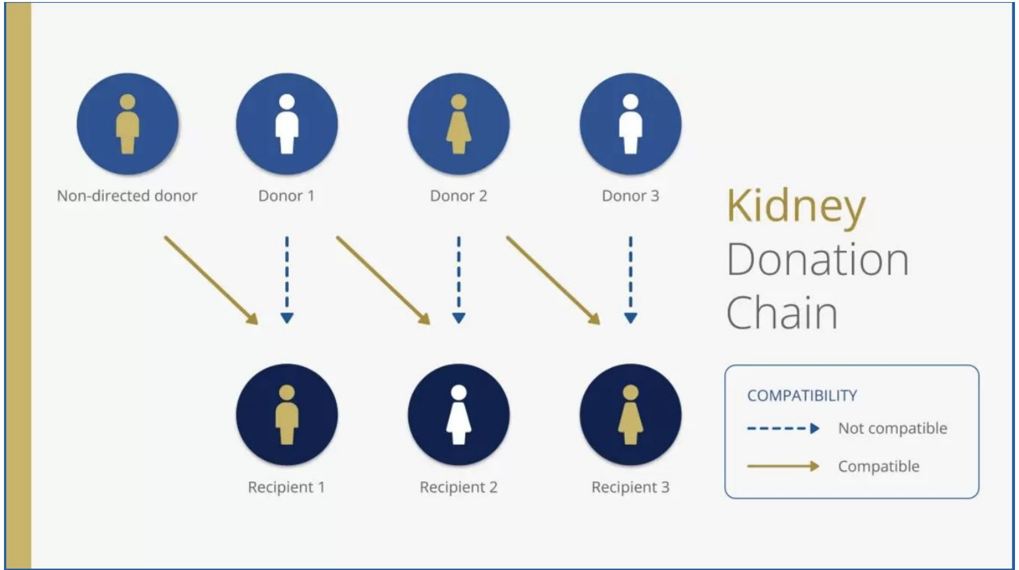


Graphs from: https://pmc.ncbi.nlm.nih.gov/articles/PMC4004760/

Final Words

Kidney Exchange: Other Structures

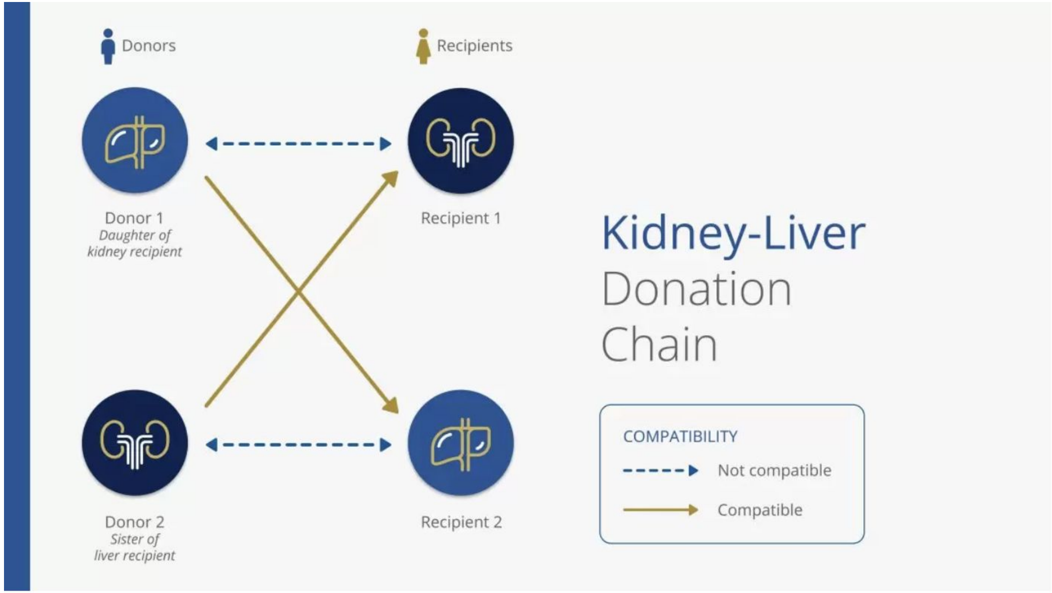
Donation chain with a non-directed donor.



A diagram of a kidney donation chain.
Credit: U.S. National Science Foundation

Kidney Exchange: Other Structures

Kidney-Liver exchange



A diagram of a kidney-liver donation chain.
Credit: U.S. National Science Foundation

Extensions To Today's Presentation

Nonlinear Programming: Objective and constraints are nonlinear in nature. Harder problems. Used in ML, Engineering Design etc.

Probabilistic Kidney Exchange: Uncertain compatibility between donor and patient.

Heuristics like local search and greedy: Quickly find feasible solutions quickly. (trade-offs between speed and optimality)

Can we just go to the summary and end this?

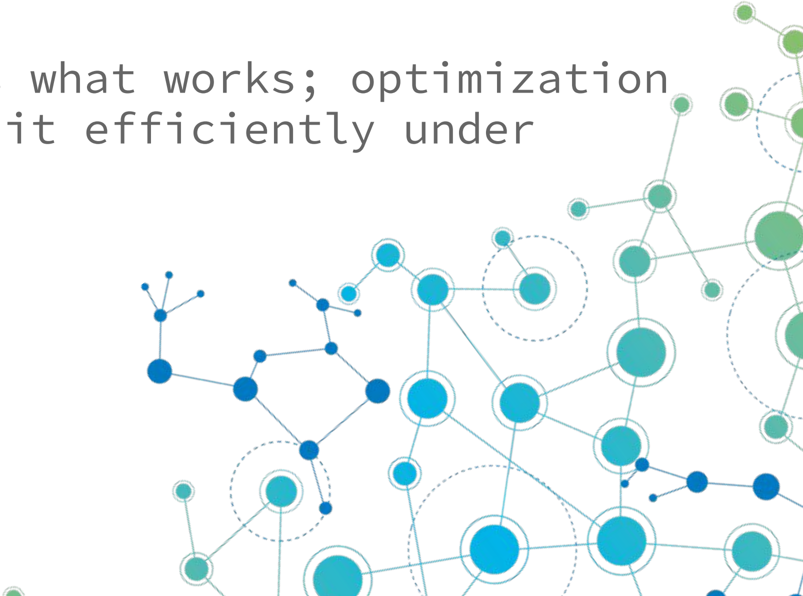
Reasons To Study Optimization

Biostatistics focuses on: Estimation, Inference, Modelling and causal effects etc.

But once we estimate effects someone has to decide things like how many vaccines to allocate, who should be matched to who etc.

Statistical evidence informs what works; optimization determines how to implement it efficiently under real-world constraints.

Please end this. My stomach is already infeasible – no more of her baked goods allowed!



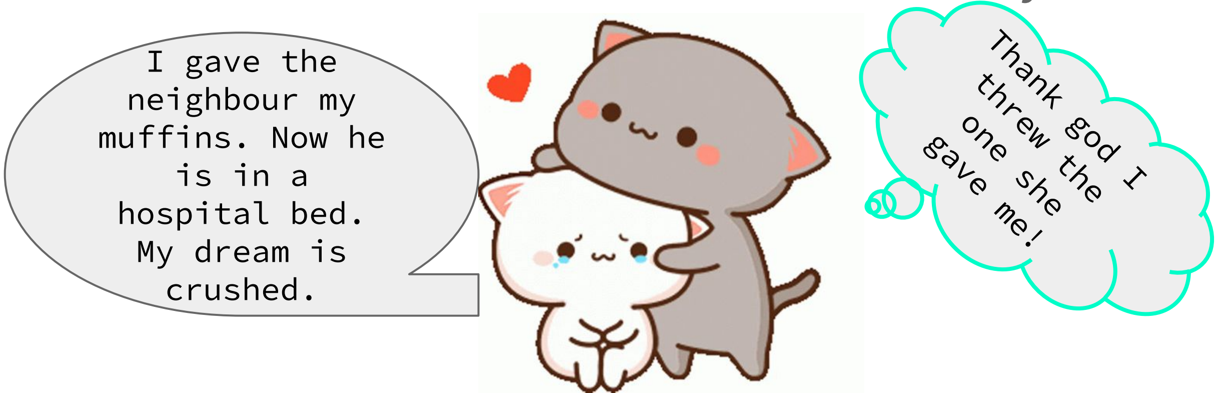
Summary

We saw the basic structure of an optimization problem: feasibility, boundedness, optimization function etc.

Became familiar with LP, IP, MIP, Multiobjectivity etc. We did some basic coding for the same on PuLP in Python.

Studied a real-world health application: Kidney exchange.

Discussed some extensions: advanced considerations beyond the “basics”.


I gave the neighbour my muffins. Now he is in a hospital bed. My dream is crushed.
Thank god I threw the one she gave me!

Scan to mark your attendance!

“An eye for an eye will make the whole world blind” - Gandhi, 19XX

“A kidney for a kidney will let the whole word CENSORED” - Vidit, 2026

Thank You!

Peach and Goma (Mochi Mochi Peach Cat) characters by Bu Jue Xiao Xiao (不觉晓晓).

References:
[1] https://medium.com/@gazalashaikh999/mixed-integer-programming-cfe0c196e875
[2] https://www.nsf.gov/news/kidney-exchange-life-saving-application-matching
[3] https://www3.diism.unisi.it/~agnetis/matchingENGnew.pdf